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REAL-TIME FORMALISM FOR POLARIZATION OPERATOR OF A CHARGED SCALAR PARTICLE IN EXTERNAL ELECTROMAGNETIC FIELD

We extend the Keldysh real-time formalism to investigate the retarded polarization operator of a charged scalar particle in a statistical system that under the action of an external field deviates to any extent from the state of thermodynamic equilibrium. We consider the peculiarities of the interaction picture that exactly accounts for external field, the so-called Furry picture, and the perturbation theory of electrodynamics for nonequilibrium processes. The interaction Hamiltonian, S-matrix, and density matrix in the external electromagnetic field are constructed. We use these quantities to define the expectation values and Green's functions. The detailed analysis of the expansion of the casual Green function up to the e^2 terms is performed. We obtain the one-loop retarded polarization operator of charged scalar particles in the coordinate representation. The desired expression is presented as the sum of three terms given by means of integral equations. Some applications of the obtained results to the problem of the energy spectra of charged particles at external conditions and further prospects are discussed.

Keywords: scalar electrodynamics, polarization operator, Keldysh formalism, finite temperature, external electromagnetic field.

Мы расширяем формализм реального времени Келдыша для исследования поляризационного оператора заряженных скалярных частиц в статистической системе, которая под действием внешнего поля может сколь угодно сильно отклоняться от состояния термодинамического равновесия. Рассмотрены особенности построения представления взаимодействия, которое точно учитывает внешнее поле, так называемая картина Фарри, и теории возмущений скалярной электродинамики для неравновесных процессов. Построен гамильтониан взаимодействия, S-матрица и матрица плотности во внешнем электромагнитном поле. С помощью последних определено среднее значение операторов физических величин и функции Грина. Проведен детальный анализ членов разложения порядка e^2 причинной функции Грина, на основе чего была найдена форма однопетлевого запаздывающего поляризационного оператора в координатном представлении. Искомое выражение является суммой трех слагаемых, каждое из которых определяется интегральным соотношением. Обсуждаются дальнейшие перспективы и некоторые приложения полученных результатов к задаче поиска спектров заряженных частиц во внешних условиях.

Ключевые слова: скалярная электродинамика, поляризационный оператор, формализм Келдыша, конечная температура, внешнее электромагнитное поле.

Ми розширюємо формалізм реального часу Келдиша для дослідження поляризаційного оператора заряджених скалярних частинок у статистичній системі, яка під дією зовнішнього поля може як завгодно сильно відхилятися від стану термодинамічної рівноваги. Розглянуто особливості побудови представлення взаємодії, яке точно враховує зовнішнє поле, так звана картина Фаррі, та теорії збурень скалярної електродинаміки для нерівноважних процесів. Побудовано гамільтоніан взаємодії, S-матриця та матриця густини у присутності зовнішнього електромагнітного поля. Останні було використано для визначення середніх від операторів фізичних величин і функції Гріна. Проведено детальний аналіз членів розвинення порядку e^2 причинної функції Гріна, на основі чого було знайдено форму однопетельного поляризаційного оператора, що запізнюється, в координатному представленні. Шуканий вираз є сумою трьох доданків, кожен з яких визначено інтегральним співвідношенням. Обговорюються подальші перспективи та деякі застосування отриманих результатів до задачі пошуку спектрів енергії частинок у зовнішніх умовах.

Ключові слова: скалярна електродинаміка, поляризаційний оператор, формалізм Келдиша, скінченна температура, зовнішнє електромагнітне поле.

Introduction

Spectra of charged particles in external electromagnetic fields at finite temperature are important objects having various applications.They allow calculation and investigation of all thermodynamic functions of many-particle systems. Usually, to investigate a spectrum, the imaginary time formalism of finite temperature field theory is applied. However, in this formalism to perform an analytic continuation and derive spectrum by means of functional equations, an expression of interest has to contain a convenient integral representation (such as Fock-Schwinger proper time representation). Because of such difficulty, within this approach, the spectrum of gluons at finite temperature in the presence of chromomagnetic fields was calculated only for some special cases [1 - 3]. We are going here to apply an alternative approach developed already by Keldysh [4] and Schwinger [5] in studies on nonequilibrium quantum statistics. This method is out of technical difficulties related with the analytic continuation.

In reality, an environment considered is realized in heavy ion collision experiments at RHIC and LHC where very strong magnetic field are generated [6, 7]. The presence of magnetic field can be a source of properties of quark-gluon plasma discovered in modern experiments. So, the working out of formalism convenient in the described background is of interest.

In the present paper, to develop formalism, we investigate the case of scalar electrodynamics. At first, we consider the Keldysh formalism in the presense of external comditions. Further the perturbative analysis of exact two-point Green function is performed and the one-loop polarization operator of a charged scalar particle is calculated.

Scalar electrodynamics at external conditions

In the present section we develop the Keldysh formalism for scalar electrodynamics in an external field considering the one-loop polarization operator of a charged scalar particle as example.

The Lagrangian of a charged spinless particle in the external field reads

$$L = (\partial_\mu - ieA_\mu - ieA_\mu^{ext})^* \varphi^+(x)(\partial_\mu - ieA_\mu - ieA_\mu^{ext})\varphi(x) - m^2\varphi^+(x)\varphi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad e = |e|, \quad g^{\mu\nu} = diag(1,-1,-1,-1) \tag{1}$$

where A_μ is potentail of radiation field, A_μ^{ext} denotes potential of external electromagnetic field, satysfying the equation of motion $\partial_\mu F_{ext}^{\mu\nu} = 0$.

We present the Lagrangian as the sum of three terms

$$L = L_0^\gamma + L_0 + L_{int}.$$

They are as follows

$$L_0 = (\partial_\mu - ieA_\mu)^* \varphi^+(x)(\partial_\mu - ieA_\mu)\varphi(x) - m^2\varphi^+(x)\varphi(x)$$

is the Lagrangian of scalar particle in the external field,

$$L_0^\gamma = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

is the Lagrangian of free radiation field,

$$L_{int} = ie\varphi^+ D_\mu \varphi \cdot A^\mu - e^2\varphi^+\varphi A^2$$

is the interaction Lagrangian where

$$\varphi^+ D_\mu \varphi = -(\partial_\mu - ieA_\mu^{ext})^* \varphi^+ \varphi + \varphi^+(\partial_\mu - ieA_\mu^{ext})\varphi .$$

We define canonical momenta and introduce interaction Hamiltonian

$$H = \int d^3x(\pi\pi^+ + ieA_0^{ext}(\pi\varphi - \pi^+\varphi^+) + (\vec{\nabla} + ie\vec{A}_{ext})\varphi^+(\vec{\nabla} - ie\vec{A}_{ext})\varphi - m^2\varphi^+\varphi +$$
$$+ ieA_0(\pi\varphi - \pi^+\varphi^+) + ie\vec{A}\varphi^+(\vec{\nabla} - ie\vec{A}^{ext})\varphi - ie(\vec{\nabla} + ie\vec{A}_{ext})\varphi^+ A_{ext}\varphi + e^2\varphi^+\varphi\vec{A}^2) + H_0^\gamma \quad (2)$$

Note the relation between interaction Lagrangian and Hamiltonian,

$$H_{int} = -L_{int} - e^2 A_0^2 \varphi^+ \varphi. \quad (3)$$

As we see, these functions differ by a non-covariant term, as is usual for time-dependent interactions [8].

Scalar electrodynamics in external fields at finite temperature

Following [4], we define the density matrix

$$i\frac{\partial\rho(t)}{\partial t} = H_{int}(t)\rho(t) - \rho(t)H_{int}(t) \quad (4)$$

with boundary conditions

$$\rho(-\infty) = \rho_0 = exp\frac{\Omega_0 - H_0(-\infty) - H_0^\gamma(-\infty)}{kT}$$

where Hamiltonians are defined according to the previous section.

The complete set of wave functions in magnetic field is the solution of the equation

$$((\hat{p}_\mu + eA^{ext})^2 - m^2)\varphi_n(x) = 0,$$
$$A_\mu^{ext} = (0,0,H\cdot x,0).$$

We split the wave functions of stationary states ϕ_n with defined momenta p_y, p_z, and energy

$$\varepsilon_n^\pm = \pm\sqrt{p_z^2 + m^2 + |e|(2n+1)},$$
$$\varphi_n = e^{-i\varepsilon_n^\pm t + ip_y y + ip_z z} f_n(x)$$

in two sets. The first set contains wave functions with positive frequencies, the second one - with negative frequencies. We denote them as $\varphi_n^\pm(x)$. Field operators φ and φ^+ are of the form

$$\varphi_0(x) = \sum_n(\varphi_n^+(x)a_n^- + \varphi_n^-(x)b_n^+),$$
$$\varphi_0^+(x) = \sum_n(\varphi_n^{+*}(x)a_n^+ + \varphi_n^{-*}(x)b_n^-),$$

and $a_n^\pm$, $b_n^\pm$ satisfy the commutation relations for Bose-operators. Field operators satisfy the equation

$$i\frac{\partial\varphi_0}{\partial t} = [\varphi_0, H_0].$$

These operators and operators of radiation field A_μ form the interaction picture of scalar electrodynamics in an external field (so-called Furry picture).

We solve the density matrix equation with S-matrix

$$S(t,-\infty) = T\,exp(-i\int_{-\infty}^{t} H_{int}(t')dt').$$

Density matrix at time t is

$$\rho(t) = S(t,-\infty)\rho(t)S^+(t,\infty).$$

In what follows we consider the statistical averages of operator products taken at various time moments. So it is convenient to carry-over the time dependence of density matrix to operators, therefore we switch into the Heisenberg picture. We set density matrix at time t = 0 accounting for all changes from the moment of switching the interaction on:

$$\rho = \rho(0) = S(0,-\infty)\rho(t)S^{+}(0,\infty).$$

Now we use the operators which at t = 0 are equal to the free ones because the density matrix is defined as $\rho(0)$

$$\varphi(x) = S(0,x)\varphi_0(x)S(x,0)$$

where, by definition,

$$S(t',t) = T\, exp\left(-i\int_t^{t'} H_{int}(t'')dt''\right) = S(t',-\infty)S^{+}(t,-\infty).$$

Statistical average of the T-products of Heisenberg operators can be transformed into the form

$$Tr(\rho A(t)B(t')...) = Tr(\rho_0 T_c(A_0(t)B_0(t')...S_c)), \tag{5}$$

where T_c stands for ordering along the line which runs from $-\infty$, passes throught the points $t, t'...$, runs to $+\infty$ and then returns back to $-\infty$.

The statistical averages of the T-products for operators of scalar and electromagnetic fields in the interaction picture

$$Tr(\rho_0 T(\varphi(x_1)\varphi(x_2)...\varphi(x_n)\varphi^{+}(x_1)\varphi^{+}(x_2)...\varphi^{+}(x_n))$$

$$Tr(\rho_0 T(A_\mu(x_1)...A_\nu(x_{2n}))$$

can be reduced to sums of the products of pairs of these operators [9].

The Green functions of scalar field φ in the interaction picture are defined as follows

$$G_0^F(t_+\vec{r},t'_+\vec{r}') = -iTr(\rho_0 T_c(\varphi_0(t_+\vec{r})\varphi_0^{+}(t'_+\vec{r}'))) = -iTr(\rho_0 T(\varphi_0(t_+\vec{r})\varphi_0^{+}(t'_+\vec{r}'))),$$

$$G_0^{\widetilde{F}}(t_-\vec{r},t'_-\vec{r}') = -iTr(\rho_0 T_c(\varphi_0(t_-\vec{r})\varphi_0^{+}(t'_-\vec{r}'))) = -iTr(\rho_0 \widetilde{T}(\varphi_0(t_-\vec{r})\varphi_0^{+}(t'_-\vec{r}'))),$$

$$G_0^{-}(t_-\vec{r},t'_+\vec{r}') = -iTr(\rho_0 T_c(\varphi_0(t_-\vec{r})\varphi_0^{+}(t'_+\vec{r}'))) = -iTr(\rho_0 \varphi_0(t_-\vec{r})\varphi_0^{+}(t'_+\vec{r}'))),$$

$$G_0^{+}(t_+\vec{r},t'_-\vec{r}') = -iTr(\rho_0 T_c(\varphi_0(t_+\vec{r})\varphi_0^{+}(t'_-\vec{r}'))) = -iTr(\rho_0 \varphi_0^{+}(t'_+\vec{r}')\varphi_0(t_+\vec{r}))$$

where t_+ stands for the point on the positive branch of integration line, t_- stands for the point on the negative one, and $\widetilde{T}$ denotes anti-time-ordering operator.

Exact Green's functions are defined as

$$G^F(t_+\vec{r},t'_+\vec{r}') = -iTr(\rho_0 T_c(\phi_0(t_+\vec{r})\phi_0^{+}(t'_+\vec{r}')S_c)),$$

$$G^{\widetilde{F}}(t_-\vec{r},t'_-\vec{r}') = -iTr(\rho_0 T_c(\phi_0(t_-\vec{r})\phi_0^{+}(t'_-\vec{r}')S_c)) \tag{6}$$

$$G^{\pm}(t_\pm\vec{r},t'_\mp\vec{r}') = -iTr(\rho_0 T_c(\varphi_0(t_\pm\vec{r})\varphi_0^{+}(t'_\mp\vec{r}')S_c)).$$

The results of this section are applied to calculate the one-loop polarization operator below.

Retarded polarization operator in order e^2

To obtain the particle energy spectrum, we investigate the poles of the full retarded Green function. One can derive it by solving the Schwinger-Dyson equation where the one-loop retarded polarization operator is taken into account. The similar method was applied to find the spectrum of QGP problem in Ref. [10].

The polarization operator and Green function are related by the Schwinger-Dyson equation

$$G(x,x') = G_0(x,x') + \int dy_1 dy_2\, G_0(x,y_1)\Pi(y_1,y_2)G(y_2,x')$$

where

$$G = \begin{pmatrix} G^F & G^+ \\ G^- & G^{\widetilde{F}} \end{pmatrix},\ G_0 = \begin{pmatrix} G_0^F & G_0^+ \\ G_0^- & G_0^{\widetilde{F}} \end{pmatrix},\ \Pi = \begin{pmatrix} \Pi^F & \Pi^+ \\ \Pi^- & \Pi^{\widetilde{F}} \end{pmatrix}.$$

Considering this equation in the order e^2, we substitute $G(y_2,x')$ to $G_0(y_2,x')$ under the sign of integration.

$$G^{(2)}(x,x') = \int dy_1 dy_2 (G^F(x,y_1)\Pi^F(y_1,y_2)G^F(y_2,x') +$$
$$+G^+(x,y_1)\Pi^-(y_1,y_2)G^F(y_2,x') + G^F(x,y_1)\Pi^+(y_1,y_2)G^-(y_2,x') +$$
$$+G^+(x,y_1)\Pi^{\widetilde{F}}(y_1,y_2)G^-(y_2,x')) . \quad (7)$$

We consider $G^F(x,x')$ in order e^2 to obtain the components of the polarization operator.

$$G_0^F(t_+\vec{r}, t'_+\vec{r}') = -iTr(\rho_0 T_c(\varphi_0(t_+\vec{r})\varphi_0^+(t'_+\vec{r}')S_c)),$$

$$S = T\, exp\left(i\int_{-\infty}^{\infty} L_{int}(t)\right),\ S = \widetilde{T}\, exp\left(-i\int_{-\infty}^{\infty} L_{int}(t)\right),$$

$$G^{(2)}(x,x') = -iTr(\rho_0 \int dy_1 dy_2 (T(\varphi_0(x)\varphi_0^+(x')\frac{i^2}{2!}L_{int}^{(1)}(y_1)L_{int}^{(1)}(y_2)) +$$
$$+\widetilde{T}(\frac{(-i)^2}{2!}L_{int}^{(1)}(y_1)L_{int}^{(1)}(y_2))T(\varphi_0(x)\varphi_0^+(x')) +$$
$$+\widetilde{T}(-iL_{int}^{(1)}(y_1))T(\varphi_0(x)\varphi_0^+(x')iL_{int}^{(1)}(y_2))) +$$
$$+\rho_0\int dy_1 \Big(T(\varphi_0(x)\varphi_0^+(x')iL_{int}^{(2)}(y_1)) + \widetilde{T}(iL_{int}^{(2)}(y_1))T(\varphi_0(x)\varphi_0^+(x'))\Big)) \quad (8)$$

Let us turn to the first term of this expression

$$-iTr\rho_0 \int dy_1 dy_2\, T(\varphi_0(x)\varphi_0^+(x')\frac{i^2}{2!}L_{int}^{(1)}(y_1)L_{int}^{(1)}(y_2)) =$$
$$= -i\frac{i^2}{2}(ie)^2\int dy_1 dy_2 (\ 2D_0^{F\mu\nu}(y_1,y_2)\cdot G_0^F(x,y_1)D^\mu G_0^F(y_1,y_2)D^\nu G_0^F(y_2,x') +$$
$$+G_0^F(x,y_1)D^\mu G_0^F(y_1,x')\cdot 2D_0^{F\mu\nu}(y_1,y_2)\cdot(-iTr(\rho_0\, T(\varphi_0(y_2)D^\nu\varphi_0^+(y_2))))) +$$
$$+G_0^F(x,x')\frac{i^2}{2}\int dy_1 dy_2 Tr(\rho_0(L_{int}^{(1)}(y_1)L_{int}^{(1)}(y_2))) . \quad (9)$$

The last expression can be presented as the sum of diagrams in Fig. 1.

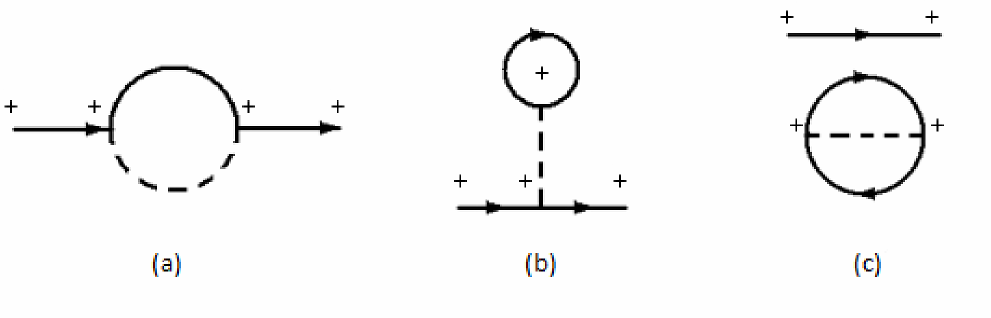

Fig.1 One-loop contributions in Π^F coming from $L^{(1)}_{int}$.

Solid and crossed lines correspond to Green's function of scalar particle and photon respectively.
Sign «+» denotes the vertex on the positive branch and «-» – on the negative one.

A type (b) diagram equals to zero in a constant homogeneous external field because the particle 4-momentum is conserved [11]. The disconnected diagrams cancel each other out. Hence, we obtain

$$\int dy_1 dy_2 G^F(x,y_1)\Pi^F_{(1)}(y_1,y_2)G^F(y_2,x') =$$

$$= ie^2 \int dy_1 dy_2 D_0^{F\mu\nu}(y_1,y_2)\cdot G_0^F(x,y_1)D^\mu G_0^F(y_1,y_2)D^\nu G_0^F(y_2,x') \quad (10)$$

where $\Pi^F_{(1)}(y_1,y_2)$ in Eq. (7), is a contribution to $\Pi^F(y_1,y_2)$ coming from $L^{(1)}_{int}(y_1)$. We denote $Tr(\rho_0\ldots)\equiv\langle\ldots\rangle_0$ for convenience.

We rewrite the next term of (8) as follows

$$-i\int dy \langle T(\varphi_0(x)\varphi_0^+(x'))iL^{(2)}_{int}(y))\rangle_0 =$$

$$= -ie^2\int dy G_0^F(x,y)G_0^F(y,x')\cdot g_{\mu\nu}D_0^{F\mu\nu}(y,y) + G_0^F(x,x')\int dy\langle T(L^{(2)}_{int}(y))\rangle_0 =$$

$$= \int dy_1 dy_2 G^F(x,y_1)\Pi^F_{(2)}(y_1,y_2)_{(2)}G^F(y_2,x') + G_0^F(x,x')\int dy\langle T(L^{(2)}_{int}(y))_0 \quad (11)$$

where

$$\Pi^F_{(2)}(y_1,y_2) = -ie^2\delta(y_1-y_2)g_{\mu\nu}\cdot D_0^{F\mu\nu}(y_1,y_2) \quad (12)$$

is the second contribution to the polarization operator $\Pi^F(y_1,y_2)$.

Let us consider the last term of the expression (8)

$$-i\int dy_1 dy_2\langle\widetilde{T}(-iL^{(1)}_{int}(y_1))T(\varphi_0(x)\varphi_0^+(x')iL^{(1)}_{int}(y_2)\rangle\rangle =$$

$$= ie^2\int dy_1 dy_2 D_0^{F\mu\nu}(y1,y2)(\ G_0^+(x,y_1)D_\mu G_0^-(y_1,x')\cdot(-i\langle T(\phi_0(y_2)D_\nu\phi_0^+(y_2)\rangle_0 +$$

$$+ G_0^F(x,y_2)D_\mu G_0^F(y_2,x')\cdot(-i\langle T(\varphi_0(y_1)D_\nu\varphi_0^+(y_1)\rangle_0 +$$

$$+ G_0^+(x,y_1)D_\mu G_0^-(y_1,y_2)D_\nu G_0^F(y_2,x') +$$

$$+ G_0^F(x,y_2)D^\mu G_0^+(y_2,y_1)D^\nu G_0^-(y_1,x')) +$$

$$+ G_0^F(x,x')\int dy_1 dy_2\langle\widetilde{T}(-iL^{(1)}_{int}(y_1))T(iL^{(1)}_{int}(y_2)\rangle_0. \quad (13)$$

We represent the obtained sum as the sum of diagrams in Fig. 2 (except for the last term).

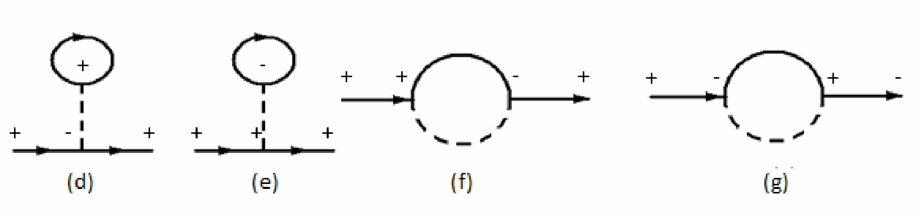

Fig. 2 One-loop contributions in Π^- and Π^+.

Diagrams of type (d) and (e) equal zero the same way as the diagram of the type (b) does. Diagrams (f) and (g) contribute to Π^- and Π^+, in accordance with Eq. (2).

Reducing the rest of terms in expression (8) by the Wick theorem

$$-i\int dy_1 dy_2 \langle \widetilde{T}(\frac{(-i)^2}{2!} L_{int}^{(1)}(y_1) L_{int}^{(1)}(y_2)) T(\varphi_0(x)\varphi_0^+(x'))\rangle_0 -$$

$$-i\int dy \langle \widetilde{T}(iL_{int}^{(2)}(y)) T(\varphi_0(x)\varphi_0^+(x'))\rangle_0, \quad (14)$$

we see that they do not contain the casual function G_0^F which depends on x or x'. Therefore they have no effect on Π^F, Π^+ and Π^- but define $\Pi^{\widetilde{F}}$. Their contribution containing vacuum diagrams is

$$G_0^F(x,x')(-i\int dy_1 dy_2 \langle \widetilde{T}(\frac{(-i)^2}{2!} L_{int}^{(1)}(y_1) L_{int}^{(1)}(y_2))\rangle_0 - i\int dy \langle \widetilde{T}(iL_{int}^{(2)}(y))\rangle_0).$$

The sum of all the vacuum loops in $G^{F(2)}(x,x')$, is a $\langle S^+ S\rangle_0$ in order e^2, and obviously equals to zero.

The retarded Green function is defined by means of statistical average over the Heisenberg density matrix

$$G^R(x,x') = i\Theta(t'-t)\langle[\varphi(x), \varphi^+(x')]\rangle.$$

It is related with the above defined Green function by the equation

$$G^R = G^F - G^+ = -G^{\widetilde{F}} + G^-$$

and satisfies the Schwinger-Dyson equation

$$G^R(x,x') = G_0^R(x,x') + \int dy_1 dy_2 G_0^R(x,y_1) \Pi^R(y_1,y_2) G^R(y_2,x')$$

where

$$\Pi^R = \Pi^F + \Pi^+ = -(\Pi^{\widetilde{F}} + \Pi^-).$$

Finally, we obtain formula for Π^+ by using Eq. (13),

$$\int dy_1 dy_2 G_0^F(x,y_1) \Pi^+(y_1,y_2) G_0^-(y_2,x') =$$

$$= -ie^2 \int dy_1 dy_2 D_0^{-\nu\mu}(y_2,y_1) \cdot G_0^F(x,y_1) D^\mu G_0^+(y_1,y_2) D^\nu G_0^-(y_2,x'). \quad (15)$$

Summing up the carried out analysis of the expansion of the full Green function, Π^F is the sum of $\Pi_{(1)}^F + \Pi_{(2)}^F$, the retarded polarization operator looks as follows

$$\Pi^R = \Pi_{(1)}^F + \Pi_{(2)}^F + \Pi^+. \quad (16)$$

Its terms are defined by means of integral equations (10), (12) and (15). In this way the polarization operator of charged scalar particle in the electromagnetic field at finite temperature in the Keldysh formalism is constructed.

Conclusions

In this paper we have considered scalar electrodynamics at external conditions. We obtained the one-loop polarization operator of a charged scalar field at a finite temperature and in an external electromagnetic field.

Given by Eq.(16), the one-loop retarded polarization operator Π^R allows to investigate spectra of charged scalar particles in different environments. For this purpose we have to substitute it into the Schwinger-Dyson equation

$$G^R(x,x') = G_0^R(x,x') + \int dy_1 dy_2\, G_0^R(x,y_1)\Pi^R(y_1,y_2)G^R(y_2,x'),$$

and study the singularity positions of the full Green function in momentum space. This problem will be considered separately.

References

1. **Skalozub V.V.** On the generation of Abelian magnetic fieds in SU(3) gluodynamics at high temperature / V.V. Skalozub, A.V. Strelchenko // Eur. Phys. J. C. – 2006. – Vol. 45, N. 1. – P. 159-178.
2. **Bordag M.** Neutral gluon polarization tensor in a color magnetic background at finite temperature / M. Bordag V. Skalozub // Phys. Rev. D. – 2007. – V.75 Issue 12. – 32 p. [arXiv:hep- th/0611256].
3. **Bordag M.** Polarization tensor of charged gluons in color magnetic background field at finite temperature / M.Bordag V. Skalozub // Phys. Rev. D. – 2008. – V.77, Issue 10. –15 p. [arXiv:0801.2306 [hep-th]].
4. **Келдыш Л. В.** Диаграммная техника для неравновесных процессов / Л.В. Келдыш // ЖЭТФ. – 1964. – Т. 47. – С. 1515-1527.
5. **Schwinger J.** Brownian motion of a quantum oscillator / J. Schwinger // J. Math. Phys. – 1961. – V. 2. – P. 407-432.
6. **Sokolov V**. Estimate of the magnetic field strength in heavy-ion collisions/ V. Sokolov, A. Illarionov, V. Toneev // Int.J.Mod.Phys.A. – 2009. – V. 24. – P. 5925-5932
7. **Voskresenskii D.N.** /Properties of a pion condensate in a magnetic field/ D.N. Voskresenskii, N. Yu. Anisimov // JETP. – 1980. – Vol. 51, No. 1. – P. 13-22.
8. **Itzykson C.** Quantum Field Theory / C. Itzykson, J.B. Zuber. – McGraw-Hill, New-York, 1998. – 05 p.
9. **Le Bellac M**. Thermal field theory / M. Le Bellac. – CUP, 1996. – 256 p.
10. **Lebedev V.V.** Spectrum of quark-gluon plasma / V.V. Lebedev, A.V.Smilga // Ann. Phys. – 1990. – V. 202 (2). – P. 229-270.
11. **Байер В.Н.** Операторный подход к квантовой электродинамике во внешнем поле: массовый оператор /В.Н. Байер, В.М. Катков, В.М. Страховенко // ЖЭТФ. – 1974. – Т. 67. С.453-470.

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