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NON-LINEAR RELAXATION IN SPATIALLY UNIFORM PLASMA

The component temperature and velocity relaxation in spatially uniform completely ionized two-component plasma is investigated on the basis of the Landau kinetic equation. The investigation is made on the basis of the generalized Chapman–Enskog method which allows us to account for the spatially homogenous relaxation processes. The local equilibrium assumption (LEA), that spatially homogenous component distribution functions (CDF) are Mawxellian ones with time-dependent component temperature and velocity, is widely used in literature. Our previous papers are devoted to the calculation of corrections to the LEA in the framework of the linear relaxation theory. In this paper we investigate the theory which is quadratic in small deviations (SD) of the component temperatures and velocities from their equilibrium values. The contributions of this theory to the CDF and to time equations for SD are calculated. At some stage of the system evolution such contributions in the leading order in small electron-to-ion mass ratio (MR) can be comparable to our corrections to the LEA of higher orders in MR which are obtained in the framework of the linear relaxation theory.

Keywords: Landau kinetic equation, generalized Chapman–Enskog method, completely ionized spatially homogenous plasma, linear relaxation, non-linear relaxation.

На основі кінетичного рівняння Ландау вивчається релаксація швидкостей та температур компонент в повністю іонізованій просторово-однорідній двокомпонентній плазмі. Дослідження проводиться на основі узагальненого методу Чепмена–Енскога, який дозволяє нам враховувати просторово-однорідні релаксаційні процеси. В літературі широко використовується наближення локальної рівноваги (НЛР) про те, що просторово-однорідні функції розподілу компонент (ФРК) є максвелівськими з залежними від часу швидкостями та температурами компонент. Наші попередні статті присвячені обчисленню поправок до НЛР в рамках лінійної теорії релаксації. В цій статті ми вивчаємо теорію, яка є квадратичною по малих відхиленнях (МВ) температур та швидкостей компонент від їх рівноважних значень. Обчислено внески, які ця теорія дає у ФКР та у часові рівняння для МВ. На деякому етапі еволюції системи ці внески у головному порядку за малим відношенням мас (ВМ) електрона й іона можуть бути порівнянними з нашими поправками до НЛР у більш високих порядках за ВМ, які обчислені в рамках лінійної теорії релаксації.

Ключові слова: кінетичне рівняння Ландау, узагальнений метод Чемпена–Енскога, повністю іонізована просторово-однорідна плазма, лінійна релаксація, нелінійна релаксація.

На основе кинетического уравнения Ландау изучается релаксация скоростей и температур компонент в полностью ионизированной пространственно-однородной двухкомпонентной плазме. Исследование проводится на основе обобщенного метода Чемпена–Энскога, который позволяет нам учитывать пространственно-однородные релаксационные процессы. В литературе широко используется приближение локального равновесия (ПЛР) о том, что пространственно-однородные функции распределения компонент (ФРК) являются максвелловскими с зависящими от времени температурами и скоростями компонент. Наши предыдущие статьи посвящены вычислению поправок к ПЛР в рамках линейной теории релаксации. В этой статье мы изучаем теорию, которая является квадратичной по малым отклонениям (МО) скоростей и температур компонент от их равновесных значений. Вычислены вклады, которые эта теория дает в ФРК и во временные уравнения для МО. На некотором этапе эволюции системы эти вклады в главном порядке по малому отношению масс (ОМ) электрона и иона могут быть сравнимы с нашими поправками к ПЛР в более высоких порядках по ОМ, которые вычислены в рамках линейной теории релаксации.

Ключевые слова: кинетическое уравнение Ландау, обобщенный метод Чемпена–Энскога, полностью ионизированная пространственно-однородная плазма, линейная релаксация, нелинейная релаксация.

1. Introduction

Component temperature and velocity relaxation in spatially homogenous completely ionized two-component electron-ion plasma is investigated on the basis of the Landau kinetic equation [1]. This equation is widely used in plasma kinetics, for example for the investigation of relaxation processes in plasma and for the investigation of transport phenomena in plasma [2–5].

The pioneering investigation of the relaxation phenomena in spatially homogenous plasma was made by Landau [1]. He derived his well-known kinetic equation and made an assumption that component distribution functions (CDF) in the spatially homogenous case are Maxwellian ones with time-dependent component temperatures. He also obtained the temperature relaxation rate in the framework of his assumption. Later in book [2] the case of different component velocities was investigated, and it was assumed that spatially homogenous CDF are Maxwellian ones with time-dependent component temperatures and velocities:

$$f_{ap}^{L} = \frac{n_a}{\left(2\pi m_a T_a\right)^{3/2}} \exp\left(-\frac{\left(p - m_a \upsilon_a\right)^2}{2 m_a T_a} \right). \tag{1}$$

This assumption is called the local equilibrium assumption (LEA). It is widely used in the literature. On the basis of the LEA the velocity relaxation rate was obtained in [2]. This assumption was used by Braginsky [3] during his calculation of plasma kinetic coefficients. Note that the results of modern investigation [4, 5] coincide with the Braginsky ones in the case of small gradients. This assumption is used in book [6], etc.

However, it can be shown [7] that the CDF of the LEA are not exact solutions of the Landau kinetic equation. They are only a solution of the leading order in the small square root of the electron-to-ion mass ratio σ,

$$\sigma = \sqrt{m_e / m_i} << 1. \tag{2}$$

In [8] corrections to the CDF and to the temperature and velocity relaxation rates are obtained in higher orders in σ.

The above-mentioned discussion is devoted to the linear relaxation theory, i.e. to the theory where only linear terms in small deviations of the component temperature and velocity from their equilibrium values are investigated. In this paper we investigate the quadratic terms in such deviations. The investigation is made on the basis of the generalization [9] of the standard Chapman–Enskog method [10]. In the general case this generalization allows us to account for the spatially homogenous relaxation of the system to the standard hydrodynamic state. In this paper we investigate only the spatially homogenous case, and thus we deal only with the spatially homogenous relaxation to the equilibrium state of the system. Quadratic relaxation contributions to the CDF and to the time equations for small deviations of the component temperature and velocity from their equilibrium values are obtained. This problem is an important one because at some stage of the system evolution these contributions in the leading order of σ can be comparable with our corrections to the LEA [8], calculated in the framework of the linear relaxation theory.

The paper is organized as follows: in Sec. 2 the basic equations of the theory are given and the generalization of the Chapman–Enskog method is described, in Sec. 3 our results for the linear relaxation theory are given, in Sec. 4 the investigation of the quadratic relaxation theory is made and in Sec. 5 the conclusions are presented.

2. Basic equations of the theory and generalization of the Chapman–Enskog method

The Landau kinetic equation [1] for spatially homogenous completely ionized electron-ion plasma has the form

$$\partial_t f_{ap}(t) = I_{ap}\left(f(t)\right),$$
$$I_{ap}\left(f\right) = -2\pi e_a^2 L \sum_c e_c^2 \frac{\partial}{\partial p_n}\int d^3p' \left\{ f_{ap}\frac{\partial f_{cp'}}{\partial p'_k} - f_{cp'}\frac{\partial f_{ap}}{\partial p_{ak}}\right\} D_{nk}\left(\frac{p}{m_a} - \frac{p'}{m_c}\right),$$
$$D_{nk}\left(u\right) \equiv \left(u^2\delta_{nl} - u_n u_l\right)\Big/ u^3\ ,\ e_e = -e\ ,\ e_i = ze \tag{3}$$

where $f_{ap}(x,t)$ is the distribution function of the a^{th} component of the plasma (a, b, $c,\ldots = e,i$), I_{ap} is the Landau collision integral, e is the elementary electric charge and L is the Coulomb logarithm.

By the standard way [11], the component particle densities, velocities and temperatures are defined in terms of the CDF:

$$n_a \equiv \int d^3p f_{ap}\ ,\ \ \pi_{an} = m_a n_a \upsilon_{an} \equiv \int d^3p f_{ap}\, p_n\ ,\ \varepsilon_a = \frac{3}{2}n_a T_a + \frac{1}{2}m_a n_a \upsilon_a^2 \equiv \int d^3p f_{ap}\varepsilon_{ap} \tag{4}$$

where π_{an} and ε_a are the a^{th} component momentum and energy density, respectively, and $\varepsilon_{ap} \equiv p^2/2m_a$.

The mass velocity υ_n and the temperature T, which describe the system in the standard hydrodynamic case, are defined [11] as

$$\pi_n \equiv \sum_a \int f_{ap} p_n d^3p = \upsilon_n \rho, \qquad \varepsilon \equiv \sum_a \int f_{ap}\varepsilon_{ap} d^3p = \frac{3}{2}\sum_a n_a T + \frac{1}{2}\sum_a m_a n_a \upsilon^2 . \tag{5}$$

In the spatially homogenous case expressions (3) – (5) give

$$\partial_t n_a = 0, \qquad \partial_t \upsilon_n = 0, \quad \partial_t T = 0, \tag{6}$$

so n_a do not depend on time and they are the equilibrium component particle densities, the velocity υ_n and the temperature T are the equilibrium temperature and velocity of the system. In what follows we use the reference frame where $\upsilon_n = 0$.

We investigate component temperature and velocity relaxation at the end of the relaxation processes, so the deviations of the electron temperature and velocity from their equilibrium values are assumed to me small and estimated by a small parameter μ:

$$\tau \equiv T_e - T, \qquad u_n \equiv \upsilon_{en} - \upsilon_n, \quad \tau \sim \mu T, \quad u \sim \mu\sqrt{T/m_e}, \qquad \mu << 1. \tag{7}$$

Equations (3) – (5) give the following time equations for τ and u_n:

$$\partial_t \tau = 2\left(Q_e - R_{en}u_n\right)\Big/ 3n_e\ , \quad \partial_t u_n = R_{en}/m_e n_e \tag{8}$$

where Q_a and R_{an} are the component energy and momentum sources:

$$R_{an} \equiv \int d^3p\, p_n I_{ap}\left(f\right), \quad Q_a \equiv \int d^3p\, \varepsilon_{ap} I_{ap}\left(f\right); \ \ \sum\nolimits_a R_{an} = 0, \quad \sum\nolimits_a Q_a = 0; \tag{9}$$

the last two expressions in (9) follow from (3).

As known [2], the reduced description parameters (RDP) of the system are the component particle densities, velocities and temperatures. On the basis of (3) – (5) it can be shown that

$$\upsilon_{in} = \upsilon_n - z\sigma^2 u_n, \quad T_i = T - z\tau - m_e z\left(1 + z\sigma^2\right)u^2 \big/ 3, \tag{10}$$

(here, the electroneutrality condition $n_e = zn_i$ is taken into account). So the ion velocity and temperature can be expressed in terms of n_a, T, υ_n, u_n and τ. Note that n_a, υ_n and T are constant equilibrium parameters, thus we can choose the system RDP as $u_n(t)$ and $\tau(t)$ and write the corresponding functional hypothesis [10]:

$$f_{ap}(t) \xrightarrow[t>>\tau_0]{} f_{ap}\left(u_n(t), \tau(t)\right), \quad \tau_0 << \tau_\upsilon << \tau_T \tag{11}$$

where τ_0 is some characteristic time which is much shorter than the velocity relaxation time τ_υ and the temperature relaxation time τ_T.

On the basis of (11) the Landau kinetic equation (3) at times $t >> \tau_0$ (we investigate the system behavior only at such times) can be rewritten:

$$\frac{2}{3n_e}\frac{\partial f_{ap}(u,\tau)}{\partial \tau}\left(Q_e - R_{en}u_n\right) + \frac{1}{m_e n_e}\frac{\partial f_{ap}(u,\tau)}{\partial u_n}R_{en} = I_{ap}\left(f(u,\tau)\right). \tag{12}$$

In what follows, equation (12) is investigated in a μ perturbation theory:

$$f_{ap} = f_{ap}^{(0)} + f_{ap}^{(1)} + f_{ap}^{(2)} + O\left(\mu^3\right), \quad f_{ap}^{(n)} \sim \mu^n. \tag{13}$$

The additional conditions for the functions $f_{ap}(u,\tau)$ follow from (4), (7) and (10):

$$\begin{gathered} n_e m_e u_n\left(\delta_{ae} - \delta_{ai}\right) = \int d^3 p p_n f_{ap}(u,\tau), \\ 3n_a T + \left(3n_e\tau + m_e n_e u^2\right)\left(\delta_{ae} - \delta_{ai}\right) = 2\int d^3 p \varepsilon_{ap} f_{ap}(u,\tau). \end{gathered} \tag{14}$$

Such a method of investigation is a generalization of the standard Chapman–Enskog method [10] similar to [9]. It allows us to account for relaxation processes in a spatially homogenous plasma. The obtained integral equations for $f_{ap}(u,\tau)$ in different orders in μ are analyzed in a σ perturbation theory with additional use of the Sonine polynomial expansion, expressions (14) are also taken into account. The σ perturbation theory is based on the standard estimates $p_{an} \sim \sqrt{m_a T}$.

Obviously the equilibrium CDF are Maxwellian ones with the equilibrium system parameters (we use the reference frame $\upsilon_n = 0$):

$$f_{ap}^{(0)} = w_{ap}, \quad w_{ap} \equiv \frac{n_a}{\left(2\pi m_a T\right)^{3/2}}\exp\left(-\beta\varepsilon_{ap}\right), \quad \beta \equiv T^{-1}, \tag{15}$$

functions (15) obey the additional conditions (14) in the leading order in μ and on the basis of (3) it can be shown that $I_{ap}(w) = 0$, so functions (15) are indeed the equilibrium solution of the Landau kinetic equation.

3. Linear relaxation theory

This section describes our results [7, 8] which are devoted to the linear relaxation theory. The functions $f_{ap}^{(1)}$ are sought as

$$f_{ap}^{(1)} = w_{ap}\left(A_a(p)\tau + B_{an}(p)u_n\right),\quad A_a(p) = A_a\left(\beta\varepsilon_{ap}\right),\quad B_{an}(p) = p_n B_a\left(\beta\varepsilon_{ap}\right), \qquad (16)$$

the last two expressions in (16) follow from the rotational invariance. It can be shown that in the framework of the linear relaxation theory the time equations for u_n, τ are

$$\begin{gathered}\left(\partial_t\tau\right)^{(1)} = -\lambda_T\tau,\quad \left(\partial_t u_n\right)^{(1)} = -\lambda_u u_n,\\ \lambda_u = \sum\nolimits_b \left\{p_l, B_{bl}(p)\right\}_{eb} \Big/ m_e n_e,\quad \lambda_T = 2\sum\nolimits_b \left\{\varepsilon_{ep}, A_b(p)\right\}_{eb} \Big/ 3n_e,\end{gathered} \qquad (17)$$

the integral brackets $\{g,h\}_{ab}$ are defined on the basis of the linearized collision integral operator $\hat{K}_{ab}$:

$$\begin{gathered}M_{ac}(p,p') \equiv \delta I_{ap} \Big/ \delta f_{cp'}\Big|_{f_p \to w_p},\quad w_{ap}K_{ac}(p,p') \equiv -w_{cp'}M_{ac}(p,p'),\\ \hat{K}_{ac}h_p \equiv \int d^3p' K_{ac}(p,p')h_{p'},\quad \left\{g_p, h_p\right\}_{ab} \equiv \int d^3p\, w_{ap} g_p \hat{K}_{ab} h_p.\end{gathered} \qquad (18)$$

The integral equations for the functions $A_a(p)$ and $B_{an}(p)$ follow from (12), (17) and (18):

$$\lambda_T A_a\left(\beta\varepsilon_{ap}\right) = \sum\nolimits_b \hat{K}_{ab} A_b\left(\beta\varepsilon_{bp}\right),\quad \lambda_u p_n B_a\left(\beta\varepsilon_{ap}\right) = \sum\nolimits_b \hat{K}_{ab} p_n B_b\left(\beta\varepsilon_{bp}\right). \qquad (19)$$

Equations (19) were investigated in [7,8] and we obtained the following results with consideration the electroneutrality condition $n_e = zn_i$:

$$\begin{gathered}A_e\left(\beta\varepsilon_{ep}\right) = -\beta S_1^{1/2}\left(\beta\varepsilon_{ep}\right) + 3\sqrt{2}z\beta(z+1)S_2^{1/2}\left(\beta\varepsilon_{ep}\right)\sigma^2 + O\left(\sigma^4\right),\\ A_i\left(\beta\varepsilon_{ip}\right) = z\beta S_1^{1/2}\left(\beta\varepsilon_{ip}\right) + 2\sqrt{2}(z+1)z^{-1}\beta S_2^{1/2}\left(\beta\varepsilon_{ip}\right)\sigma^3 + O\left(\sigma^4\right),\\ \lambda_T = \left(2/3 - \sigma^2 - 3\sqrt{2}z\sigma^2\right)\Lambda\sigma^2 + O\left(\sigma^5\right),\quad \Lambda = 2^{5/2}z^2(z+1)n_i e^4 L\sqrt{\pi}m_e^{-1/2}T^{-3/2},\\ B_e\left(\beta\varepsilon_{ep}\right) = \beta - 3z\beta(2z-1)\left(3z+4\sqrt{2}\right)^{-1} S_1^{3/2}\left(\beta\varepsilon_{ep}\right)\sigma^2 + O\left(\sigma^4\right),\\ B_i\left(\beta\varepsilon_{ip}\right) = -z\beta\sigma^2 - 3z\beta S_1^{3/2}\left(\beta\varepsilon_{ip}\right)\sigma^4/5 + O\left(\sigma^5\right),\\ \lambda_u = \left(\frac{4}{3} + \frac{4z-2}{3}\sigma^2 - 6z\frac{2z-1}{3z+4\sqrt{2}}\sigma^2\right)\Psi + O\left(\sigma^4\right),\ \Psi = \frac{\sqrt{2\pi}n_i z^2 e^4 L}{\sqrt{m_e}T^{3/2}}\end{gathered} \qquad (20)$$

where $S_\alpha^n(x)$ are orthogonal Sonine polynomials:

$$S_\alpha^n(x) \equiv \frac{1}{n!}e^x x^{-\alpha}\frac{d^n}{dx^n}\left(e^{-x}x^{\alpha+n}\right),\ \int_0^\infty e^{-x}x^\alpha S_\alpha^n(x)S_\alpha^{n'}(x)dx = \frac{\Gamma(n+\alpha+1)}{n!}\delta_{nn'}. \qquad (21)$$

Expressions (20) in the leading order in σ coincide with the well-known LEA results [1, 2], but we obtained corrections to these results in higher orders in σ. Due to the corrections to the CDF we refined the σ^4 term of λ_T and the σ^2 term of λ_u in comparison with the LEA.

4. Quadratic relaxation theory

This section is devoted to quadratic contributions in μ to the CDF and to the time equations for τ and u_n. At some stage of the system evolution such contributions in the leading order in σ can be comparable with our corrections to the LEA from the previous section, because although those corrections were obtained in the linear theory in μ, they are corrections of higher orders in σ. The functions $f_{ap}^{(2)}$ are sought as

$$f_{ap}^{(2)} = w_{ap}\left(A_a^{\tau\tau}(p)\tau^2 + A_{an}^{\tau u}(p)\tau u_n + A_{anl}^{uu}(p)u_n u_l\right),\quad A_a^{\tau\tau}(p) = A_a^{\tau\tau}\left(\beta\varepsilon_{ap}\right),$$
$$A_{an}^{\tau u}(p) = p_n A_a^{\tau u}\left(\beta\varepsilon_{ap}\right),\quad A_{anl}^{uu}(p) = \delta_{nl}A_{aA}^{uu}\left(\beta\varepsilon_{ap}\right) + h_{nlp}A_{aB}^{uu}\left(\beta\varepsilon_{ap}\right) \qquad (22)$$

where $h_{nlp} \equiv p_n p_l - p^2\delta_{nl}/3$. The structure of the functions $A_a^{\tau\tau}(p)$, $A_{an}^{\tau u}(p)$ and $A_{anl}^{uu}(p)$ follows from the rotational invariance.

The LEA results for $f_{ap}^{(2)}$ can be obtained by the use of the Taylor expansion for the LEA CDF (1):

$$A_e^{\tau\tau}(p) = \beta^2 S_2^{1/2}\left(\beta\varepsilon_{ep}\right),\quad A_i^{\tau\tau}(p) = z^2\beta^2 S_2^{1/2}\left(\beta\varepsilon_{ip}\right),$$
$$A_{enl}^{uu}(p) = -m_e\beta\delta_{nl}S_1^{1/2}\left(\beta\varepsilon_{ep}\right)/3 + \beta^2 h_{nlp}/2,\quad A_{en}^{\tau u}(p) = -\beta^2 S_1^{3/2}\left(\beta\varepsilon_{ep}\right)p_n,$$
$$A_{inl}^{uu}(p) = m_e z\beta S_1^{1/2}\left(\beta\varepsilon_{ip}\right)\delta_{nl}/3 + z^2\beta^2 h_{nlp}\sigma^4/2,\ A_{in}^{\tau u}(p) = -z^2\beta^2 S_1^{3/2}\left(\beta\varepsilon_{ip}\right)p_n\sigma^2. \qquad (23)$$

On the basis of (8), (16), (17) and (22) it can be shown that the quadratic contributions to the time equations for τ and u_n are

$$\left(\partial_t u_n\right)^{(2)} = G_{nl}^{u\tau}\tau u_n/m_e n_e,\ \left(\partial_t\tau\right)^{(2)} = 2\left(G^{\tau\tau}\tau^2 + G_{nl}^{uu}u_n u_l\right)/3n_e + 2m_e\lambda_u u^2/3;$$
$$\Theta_{aclp}\left(h_p, g_p\right) \equiv \int d^3p' w_{ap}w_{cp'}\left(h_{ap}\frac{\partial g_{cp'}}{\partial p_k'} - h_{cp'}\frac{\partial g_{ap}}{\partial p_k} + g_{ap}\frac{\partial h_{cp'}}{\partial p_k'} - g_{cp'}\frac{\partial h_{ap}}{\partial p_k}\right)\times$$
$$\times D_{lk}\left(p/m_a - p'/m_c\right),\quad V_{acl}\left(h_p, g_p\right) \equiv \int d^3p\,\Theta_{aclp}\left(h_p, g_p\right),$$
$$W_{ac}\left(h_p, g_p\right) \equiv \int d^3p\,p_l\Theta_{aclp}\left(h_p, g_p\right)/m_a,$$
$$G_{nl}^{u\tau} = 2\pi e^2 L\sum\nolimits_c e_c^2\left(V_{ecl}\left(1, A_n^{u\tau}(p)\right) + V_{ecl}\left(A(p), B_n(p)\right)\right);$$
$$G^{\tau\tau} = 2\pi e^2 L\sum_c e_c^2\left(W_{ec}\left(1, A^{\tau\tau}(p)\right) + W_{ec}\left(A(p), A(p)\right)/2\right),$$
$$G_{nl}^{uu} = 2\pi e^2 L\sum\nolimits_c e_c^2\left(W_{ec}\left(1, A_{nl}^{uu}(p)\right) + W_{ec}\left(B_n(p), B_l(p)\right)/2\right). \qquad (24)$$

The integral equations for the functions $A_a^{\tau\tau}(p)$, $A_{an}^{\tau u}(p)$ and $A_{anl}^{uu}(p)$ follow from (12), (16), (17) and (24):

$$2A_a\left(\beta\varepsilon_{ap}\right)G^{\tau\tau}/3n_e - 2\lambda_T A_a^{\tau\tau}\left(\beta\varepsilon_{ap}\right) = -\sum\nolimits_c \hat{K}_{ac}A_c^{\tau\tau}\left(\beta\varepsilon_{cp}\right) -$$
$$-\pi e_a^2 L w_{ap}^{-1}\sum\nolimits_c e_c^2\,\partial\Theta_{acnp}\left(A(p), A(p)\right)/\partial p_n,$$
$$B_{an}(p)G_{ln}^{u\tau}/m_e n_e - \left(\lambda_u + \lambda_T\right)A_{al}^{u\tau}(p) = -\sum\nolimits_c \hat{K}_{ac}A_{cl}^{u\tau}(p) -$$
$$-2\pi e_a^2 L w_{ap}^{-1}\sum\nolimits_c e_c^2\,\partial\Theta_{acnp}\left(A(p), B_l(p)\right)/\partial p_n,$$

$$-2\lambda_u A^{uu}_{anl}(p) + A_a(p)\left(2G^{uu}_{nl}/3n_e + 2m_e\lambda_u\delta_{nl}/3\right) = -\sum_c \hat{K}_{ac} A^{uu}_{cnl}(p) - \\ -\pi e_a^2 L w_{ap}^{-1}\sum_c e_c^2\, \partial\Theta_{acmp}\left(B_n(p), B_l(p)\right)/\partial p_m\,, \qquad (25)$$

These equations are investigated in a σ perturbation theory with additional use of the Sonine polynomial expansion:

$$A^{\tau\tau}_a\left(\beta\varepsilon_{ap}\right) = \sum_{n,s\geq 0} g^{\tau\tau(n)}_{as} S_s^{1/2}\left(\beta\varepsilon_{ap}\right), \quad A^{\tau u}_a\left(\beta\varepsilon_{ap}\right) = \sum_{n,s\geq 0} g^{\tau u(n)}_{as} S_s^{3/2}\left(\beta\varepsilon_{ap}\right), \\ A^{uu}_{uA}\left(\beta\varepsilon_{ap}\right) = \sum_{n,s\geq 0} g^{uu(n)}_{asA} S_s^{1/2}\left(\beta\varepsilon_{ap}\right), \quad A^{uu}_{uB}\left(\beta\varepsilon_{ap}\right) = \sum_{n,s\geq 0} g^{uu(n)}_{asB} S_s^{5/2}\left(\beta\varepsilon_{ap}\right) \qquad (26)$$

where $g_s^{(n)} \sim \sigma^n$. Expressions (14) in the second order in μ are the additional conditions to equations (25). We restrict ourselves to the calculation of the leading order in σ for the functions (26). The following results for the component CDF are obtained:

$$A^{\tau\tau}_a(p) = \beta^2\left(z^2\delta_{ai} + \delta_{ae}\right) S_2^{1/2}\left(\beta\varepsilon_{ap}\right) + O(\sigma), \\ A^{uu}_{enl}(p) = -m_e\beta S_1^{1/2}\left(\beta\varepsilon_{ep}\right)\delta_{nl}/3 + h_{nlp} g^{uu(0)}_{e0B} + O(\sigma), \\ g^{uu(0)}_{e0B} = \beta^2\left(1 - 2\sqrt{2}z/3\right)^{-1}/2\,,\ A^{\tau u}_{el}(p) = -\beta^2\left(1 + 3z/4\sqrt{2}\right)^{-1} p_l S_1^{3/2}\left(\beta\varepsilon_{ep}\right) + O(\sigma), \\ A^{uu}_{inl}(p) = \left(m_e z\beta S_1^{1/2}\left(\beta\varepsilon_{ip}\right)/3 + O(\sigma)\right)\delta_{nl} + h_{nlp}\left(g^{uu(4)}_{i0B} + O\left(\sigma^5\right)\right), \\ g^{uu(4)}_{i0B} = -2z g^{uu(0)}_{e0B}\sigma^4/5 + z^2\beta^2\sigma^4/2\,,\ A^{\tau u}_{il}(p) = -z^2\beta^2 p_l S_1^{3/2}\left(\beta\varepsilon_{iP}\right)\sigma^2 + O\left(\sigma^2\right). \qquad (27)$$

Here the results for the delta-terms of $A^{uu}_{anl}(p)$ and for $A^{\tau u}_{il}(p)$ in the leading order in σ are exact solutions and all the other results in (27) are the results of the one-polynomial approximation. As seen, only the results for $A^{\tau\tau}_a(p)$, $A^{\tau u}_{il}(p)$ and the delta-terms of $A^{uu}_{anl}(p)$ coincide with the LEA (23), so in the framework of the quadratic relaxation theory the LEA is not a solution of the Landau kinetic equation even in the leading order in σ.

The quadratic contributions to the time equations can be calculated on the basis of (20)–(22), (24) and (27):

$$\left(\partial_t\tau\right)^{(2)} = \left(\Lambda\beta\sigma^2 + O\left(\sigma^3\right)\right)\tau^2 + \left(8m_e\Psi/9 + O(\sigma)\right)u^2\,, \\ \left(\partial_t u_l\right)^{(2)} = \left(2\left(1 + 3z/4\sqrt{2}\right)^{-1}\beta\Psi + O(\sigma)\right)\tau u_l\,. \qquad (28)$$

Although we restrict ourselves to the leading order in σ, equations (24) and (25) allow us to obtain results in higher orders in σ because they contain the corrections (20) of higher orders in σ to the well-known results of the linear relaxation theory.

5. Conclusions

Component temperature and velocity relaxation in spatially homogenous completely ionized two-component electron-ion plasma is investigated on the basis of the Landau kinetic equation. The relaxation is investigated up to the second order of smallness in the small relaxation parameters τ, u_n.

Contributions quadratic in τ, u_n to the CDF (27) and to the time equations for τ, u_n (28) are investigated. Integral equations (IE) for each part of the CDF are obtained.

Although we restrict ourselves to the calculation of these contributions in the leading order in σ, the obtained IE allow us to obtain results in higher orders in σ because they contain the corrections (20) of higher orders in σ to the LEA results in the linear relaxation theory.

It is shown that even in the leading order in σ the LEA CDF are not solutions of the Landau kinetic equation in the framework of the quadratic relaxation theory. The results for the delta-terms of $A^{uu}_{anl}(p)$ and for $A^{\tau u}_{il}(p)$ are exact solutions of the Landau kinetic equation and they coincide with the LEA. The other parts of the CDF are calculated in the one-polynomial approximation. The one-polynomial result for $A^{\tau\tau}_{a}(p)$ coincide with the LEA, but the result for $A^{\tau u}_{el}(p)$ and the results for the traceless terms of $A^{uu}_{anl}(p)$ do not coincide with the LEA even in the leading order in σ and in the one-polynomial approximation. It can also be shown that the IE for the functions $A^{\tau\tau}_{anl}(p)$ are Fredholm IE of the first kind. The results for the time equations are obtained on the basis of the results for the CDF. As seen (28), the quadratic terms decrease the relaxation rates of the temperature and velocity relaxation.

At some stage of the system evolution these corrections can be comparable with the corrections (20) of higher orders in σ which are obtained in the framework of the linear relaxation theory, so the problem under consideration is an important one. The results of the paper can be taken into account in the investigation of the relaxation of plasma to the standard hydrodynamic state in the spatially non-uniform case.

References

1. **Landau, L. D.** Kineticheskoe uravnenie v sluchae kulonovskogo vzaimodeystviya [Text] / L. D. Landau // ZhETF, – 1937. – Vol. 7, – P. 203 – 209.

2. **Rukhadze A.A.** Principles of Plasma Electrodynamics [Text] / A. A. Rukhadze, A. F. Alexandrov, L. S. Bogdankevich. –2nd edition. – Moscow: URSS, 2013. – 504 p.

3. **Braginsky, S.I.** Javlenija perenosa v polnost'ju ionizirovannoj dvuhtemperaturnoj plazme [Text] / S.I.Braginsky // ZhETF – 1957. – Vol. 33, – P. 459 – 472.

4. **Brantov A.V.** Nonlocal Transport in Hot Plasma. Part I [Text] / A.V. Brantov, V. Yu. Bychenkov // Plasma Physics Reports. – 2013. – Vol. 39, No. 9. – P. 698 – 744.

5. **Brantov A.V.** Nonlocal Transport in Hot Plasma. Part II [Text] / A.V. Brantov, V.Yu. Bychenkov // Plasma Physics Reports. – 2014. – Vol. 40, No. 7. – P. 505 – 563.

6. **Zhdanov V.M.** Processy perenosa v mnogokomponentnoj plazme [Text] / V. M. Zhdanov. – Moscow: Fizmatlit, 2009. – 280p.

7. **Sokolovsky A.I.** To the Landau theory of relaxation phenomena in plasma [Text] / A. I. Sokolovsky, V. N. Gorev, Z. Yu. Chelbaevsky // Problems of Atomic Science and Technology. – 2012. – Series: Nuclear Physics Investigations, No. 1. (57) – P. 230 – 234.

8. **Gorev V.N.** Relaxation Phenomena in a Homogenous Plasma [Text] / V. N. Gorev, A. I. Sokolovsky // Conference Proceedings of the International Conference on Mathematical Methods in Electromagnetic Theory. – Dnipropetrovsk, 2014. – P. 221 – 224.

9. **Gorev V.N.** Investigation of nonequilibrium processes in vicinity of hydrodynamic states [Text] / V.N. Gorev, A.I. Sokolovsky // Proceedings of Institute of Mathematics of NAS of Ukraine, Kyiv. – 2014. – Vol. 11, No. 1. – P. 67 – 92.

10. **Colangeli M.** From Kinetic Models to Hydrodynamics: Some Novel Results [Text] / M. Colangeli – New York: Springer, 2013. – 104 p.

11. **Silin V. P.** Vvedenie v kineticheskuju teoriju gazov [Text] / V. P. Silin. – 3rd edition. – Moscow: URSS, 2013. – 344 p.

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